Now that we have laid the groundwork for permutations and combinations, probability will be a piece of cake. We just need to build up on what we have already learned.

The single most important concept in probability is the following:

The probability of an event A is calculated as P(A) = No. of outcomes when A occurs/Total no. of outcomes.

In this post, we will just extend the combinatorics concepts and apply them to probability. Let me explain how we will do it using some examples.

Example 1: Six friends live in the city of Monrovia. There are four natural attractions around Monrovia – a waterfall, a safari, a lake and some caves. The friends decide to take a vacation together at one of these attractions. To select the attraction, each one of them votes for one of the attractions. What is the probability that each one of them votes for the safari?

Solution: Here, A, the event for which we want to find the probability is 'all six friends vote for the safari'

P(A) = No of ways in which all six can vote for the safari/Total no. of ways in which they can vote.

What is the no. of ways in which all six vote for the safari? Only one way. They all vote for the safari!

What is the no. of ways in which the friends can vote? Say, the friends are A, B, C, D, E and F. A can vote in 4 ways. B can vote in 4 ways. C can vote in 4 ways and so on… Total no of ways in which the 6 friends can vote = 4*4*4*4*4*4 = 4^6 (Using our old friend, the basic counting principle). We discussed this concept in our post onUnfair Distributions.

Therefore, P(A) = 1/(4^6)

Finding this probability involved the use of the concepts we have already learned in combinatorics. I hope you see that it is quite simple and straight forward. Let's tweak this example a little to make it slightly complicated.

Example 2: Six friends live in the city of Monrovia. There are four natural attractions around Monrovia – a waterfall, a safari, a lake and some caves. The friends decide to take a vacation together at one of these attractions. To select the attraction, each one of them votes for one of the attractions. What is the probability that each one of them votes for the same attraction?

Solution: Here, A, the event for which we want to find the probability is 'all six friends vote for the same attraction'. We don't have a specific attraction given to us. So the selected attraction could be any one of the given four.

P(A) = No of ways in which all six can vote for the same attraction/Total no. of ways in which they can vote.

What is the no. of ways in which all six vote for the same attraction? They could all vote for the waterfall or for the safari or for the lake or for the caves. All of them can vote for the same attraction in 4 ways.

What is the no. of ways in which the friends can vote? As we saw in question no. 1, total no of ways in which the 6 friends can vote = 4*4*4*4*4*4 = 4^6

Therefore, P(A) = 4/(4^6) = 1/(4^5)

Now, let's make the question even trickier.

Example 3: Six friends live in the city of Monrovia. There are four natural attractions around Monrovia – a waterfall, a safari, a lake and some caves. The friends decide to take a vacation together at one of these attractions. To select the attraction, each one of them votes for one of the attractions. What is the probability that each attraction gets at least one vote?

Solution: Here, A, the event for which we want to find the probability is ‘each attraction gets at least one vote’.

P(A) = No of ways in which each attraction gets at least one vote /Total no. of ways in which the friends can vote.

Each attraction should get at least one vote. 6 votes can be divided among 4 attractions in the following ways: (1, 1, 1, 3) and (1, 1, 2, 2)

Case 1: (1, 1, 1, 3)

First, we select the attraction that will get 3 votes in 4 ways (= 4C1)

Now, we can select the 3 people who will vote for this attraction in 6*5*4/3! = 20 ways (= 6C3 )

The other 3 votes will be distributed among the other 3 attractions in 3! = 6 ways

The 6 people could vote for the 4 attractions in this case in 4*20*6 = 480 ways

Case 2: (1, 1, 2, 2)

Let’s select the two attractions that will get 2 votes each in 4*3/2! = 6 ways (= 4C2). Say we select caves and waterfall.

Now, we can select the 2 people who will vote for one of the selected attractions in 6*5/2! = 15 ways (= 6C2)

We can select the other 2 people who will vote for the other selected attraction in 4*3/2! = 6 ways (= 4C2)

The other 2 votes will be distributed among the other 2 attractions in 2! = 2 ways

The 6 people could vote for the 4 attractions in this case in 6*15*6*2 = 1080 ways

Total number of ways in which 6 votes can be distributed among 4 attractions such that each attraction gets at least one vote = 480 + 1080 = 1560 ways

As we saw in the questions above, the total no. of ways in which the friends can vote = 4^6

Therefore, P(A) = 1560/(4^6)

I hope you see that probability is just an extension of combinatorics. Some important concepts in Probability e.g. Independent events, mutually exclusive events, dependent events etc are discussed in detail in your Combinatorics and Probability book. Go through that theory before next Monday. We will discuss some tricky questions related to those concepts next week.